



DEVELOPMENT OF REPLACEMENT MODEL CONSIDERING NON-LINEAR PATTERN FOR RUNNING COST AND INFLATION

Revanth Sai Yayavaram

Dr. S. Gajanana

Kshetty Rahul

Abstract

In today's vying business world, time plays a major role and everyone desires to get a swift service within a minimal waiting time. During this process, it is uncertain about the machine's working as it may stop due to its failure or some other reason, which is not an ideal situation. So, as to minimize this, an operational researcher strives to develop a replacement model considering all the parameters affecting it to find out the optimum replacement period. The replacement problems are concerned with the situation that arises on a decrease in the efficiency of the item, failure or breakdown. The problem of replacement is to identify the best policy to determine the ideal replacement time which is most economical. In the current paper, different replacement models have been developed considering macroeconomic variable inflation and maintenance expenditure in a non-linear trend.

Keywords: Inflation, Replacement, Running cost, Monetary value, Salvage.

1 INTRODUCTON

Inflation is a quantitative measure of rate at which average cost increases over a period of time. Precisely, it is the gradual rise in the level of prices

whereas, monetary value is a consequence of inflation where, if there is a rise in inflation monetary value automatically goes down. Nominal interest rate, according to Irving Fisher, is related to inflation and real interest rate. It is given by the difference between nominal interest rate and inflation that is expected or actual.

$$(1+i) = (1+R)(1+\phi)$$

Which is equivalent to $i = R + \phi (1+R)$

If i increases $x\%$ then nominal interest rate increases by more than $x\%$ and from this it is evident that with help of any two factors the other one can be determined.

Usage of any item for a period will reduce its efficiency, thereby resulting in failure or breakdown which ultimately leads to higher maintenance cost. As a matter of fact, failure is undetermined and hence an operational researcher always tends to find out the optimum replacement period by considering maintenance costs, monetary value, etc. for each period. Replacement model can be categorized into two types and they are: Replacement of assets that deteriorate with time, Replacement of assets due to sudden failure. As mentioned earlier, through passage of time the item or the asset may be subjected to wear and tear and will reach a point where maintenance cost will be prohibitively large that the replacement will be a better option. Not only maintenance cost but also monetary value, salvage value and inflation are taken into consideration to address the optimum time to replace the asset. Forecasts are estimates of the occurrence, timing or magnitude of future events and forecasting is a technique to estimate them. Thus, no matter

what the planning is (financial or productive etc) managers have to employ the tool of forecasting.

2 NOTATIONS/LITERATURE

- i. n = time period
- ii. ϕ = inflation
- iii. $F(n)$ = forecasted maintenance cost equation
- iv. α, β, k, p = coefficients
- v. R = real interest rate
- vi. v = present worth factor
- vii. i = nominal interest rate
- viii. C = cost
- ix. S = salvage value

3 FORECASTING OF INFLATION CONSIDERING NON-LINEAR PATTERN

In the current work, it is assumed that the inflation, a macroeconomic variable follows the non-linear pattern as given in equation (1) and forecasting of inflation is carried out for future periods.

$$\phi(n) = \alpha_0 + \alpha_1 n + \alpha_2 / n \quad (1)$$

here $\phi(n)$ is the inflation, 'n' is a variable and $\alpha_0, \alpha_1, \alpha_2$ are coefficients.

Table 1: Inflation Data For 10 Periods.

Period	1	2	3	4	5
Inflation	1.8	2.3	2.7	3.2	3.6

Period	6	7	8	9	10
Inflation	4.1	4.5	5.0	5.4	5.9

The following set of equations are considered in getting the values of coefficients

$\alpha_0, \alpha_1, \alpha_2$

$$\Sigma \phi = n\alpha_0 + \alpha_1 \Sigma n + \alpha_2 \Sigma (1/n) \quad (2)$$

$$\Sigma \phi n = \alpha_0 \Sigma n + \alpha_1 \Sigma n^2 + \alpha_2 \quad (3)$$

$$\Sigma \phi n^2 = \alpha_0 \Sigma n^2 + \alpha_1 \Sigma n^3 + \alpha_2 \Sigma n \quad (4)$$

By solving these equations, we get the following values for the coefficients

$$\alpha_0 = 1.429, \alpha_1 = 0.449, \alpha_2 = 0.003$$

the final regression equation for inflation is

$$\phi(n) = 1.429 + 0.449n + 0.003/n$$

table 2 indicates forecasted inflation values for the next ten years.

Table 2: Predicted Inflation Data

Period	1	2	3	4	5
Inflation	6.3	6.8	7.2	7.7	8.1

Period	6	7	8	9	10
Inflation	8.6	9.0	9.5	9.9	10.4

4 DEVELOPMENT OF REGRESSION MODEL WITH LOGARITHMIC TREND MAINTENANCE COST AND LINEAR TREND SALVAGE VALUE

Equipment replacement techniques are mainly concerned with the cases which arise when the efficiency of the equipment or item decreases, failure or breakdown happens. The failure of the item may be sudden or gradual. In general, capital equipment failures may fall in the repairable situation (minor repair or major repair). The objective of the replacement philosophy is to identify the best policy of an age at which the replacement is economical, instead of continuing at increase running or maintenance cost. The fundamental aim of replacement models is to drive the company to minimize the cost and obtain a better quality of the products and enhance customer's satisfaction. The problem of replacement can be applied in the case of both men and machines. In the present paper, it is assumed that the maintenance of a machine varies non-linearly (partly linear and partly logarithmic) with the governing equation as mentioned below:

$$F(n) = \beta_0 + \beta_1 n + \beta_2 \log(n) \quad (5)$$

Here $F(n)$ indicates the running cost and n is the time period $\beta_0, \beta_1, \beta_2$ are the coefficients. The model can be fit by obtaining the values of regression coefficients using following set of equations.

$$\Sigma F = n\beta_0 + \beta_1 \Sigma n + \beta_2 \Sigma \log(n) \quad (6)$$

$$\Sigma Fn = \beta_0 \Sigma n + \beta_1 \Sigma n^2 + \beta_2 \Sigma n \log(n) \quad (7)$$

$$\Sigma Fn^2 = \beta_0 \Sigma n^2 + \beta_1 \Sigma n^3 + \beta_2 \Sigma n^2 \log(n) \quad (8)$$

The following yearly maintenance cost (in rupees) for the machine is used to get trend equation of maintenance. The machine is procured at a cost of $C = \text{Rs. 1000}$. The nominal

interest rate is assumed to be $i = 15\%$.

Table 3: Maintenance Data Of Machine

Period	1	2	3	4	5
Inflation	187.5	259.5	309	388.5	451.5

Period	6	7	8	9	10
Inflation	514.5	577.5	639	700.5	762

By solving the above equations, the following values for coefficients are obtained.

$$\beta_0 = 124.57, \beta_1 = 60.89, \beta_2 = 29.73$$

this is the final regression equation for maintenance cost (forecasted) obtained is $F(n) = 124.57 + 60.89n + 29.73\log(n)$

It is also assumed that resale or salvage value of the item follows a linear trend with a governing relation.

The following set of equations and yearly data from table 4 are used to determine the values of the coefficient's 'k', 'p'.

$$S = k - pn \quad (9)$$

$$\Sigma S = nk - p\Sigma n \quad (10)$$

$$\Sigma Sn = k\Sigma n - p\Sigma n^2 \quad (11)$$

Table 4: Salvage Data Of Machine

Period	1	2	3	4	5
Inflation	540	510	480	450	420

Period	6	7	8	9	10
Inflation	390	360	330	300	270

By solving these equations

$$k = 570, p = 30$$

the final regression equation obtained for salvage value is $S(n) = 570 - 30n$.

5 REPLACEMENT MODELS

In the current paper different cases of replacements are discussed by considering three major deciding parameters (inflation, monetary value, salvage value).

5.1 Replacement Decision Considering Salvage Value

5.1.1 Case-1: Considering Inflation And Monetary Value

Taking inflation and monetary value into consideration, average annual cost for each period is attained and using this average annual cost replacement period is found out.

TABLE 5: Calculation of average annual cost with inflation and MONETARY VALUE taken in consideration.

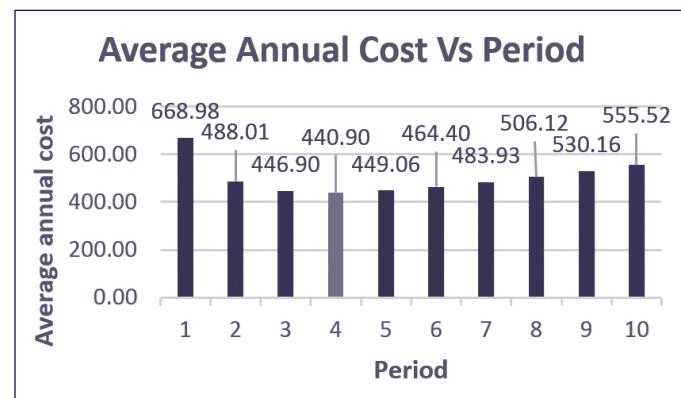
Period (n) (1)	Inflation ($\phi(n)$) (2)	Real interest rate (R) (3)	Present worth factor (v) (4)	Discount factor (v^{n-1}) (5)
1	1.88	4.55	0.96	1.00
2	2.33	3.81	0.96	0.96
3	2.78	3.24	0.97	0.94
4	3.23	2.79	0.97	0.92
5	3.67	2.42	0.98	0.91
6	4.12	2.12	0.98	0.90
7	4.57	1.87	0.98	0.89
8	5.02	1.66	0.98	0.89
9	5.47	1.47	0.99	0.89
10	5.92	1.31	0.99	0.89

Dividing Dis- count Factor (Σv^{n-1}) (6)	Forecasted Maintenance Cost (F) (7)	Maintenance cost with MON- ETARY VALUE ($F v^{n-1}$) (8)	Cumulative Maintenance Cost ($\Sigma F v^{n-1}$) (9)
1.00	185.46	185.46	185.46
1.96	255.30	245.94	431.40
2.90	321.42	301.59	732.99
3.82	386.03	355.48	1088.47
4.73	449.80	408.73	1497.19
5.63	513.04	461.89	1959.08
6.53	575.92	515.29	2474.38
7.42	638.54	569.14	3043.52
8.31	700.95	623.58	3667.10
9.20	763.20	678.70	4345.79

Salvage Value (S) (10)	Salvage with MONETARY VALUE (S *(v^n)) (11)	Total Annual Cost (TC = C - (S *(v^n)) + $\SigmaF v^{n-1}$) (12)	Average Annual Cost (AC = TC/ Σv^{n-1}) (13)
540.00	516.48	668.98	668.98
510.00	473.28	958.12	488.01
480.00	436.26	1296.73	446.90
450.00	403.15	1685.31	440.90
420.00	372.62	2124.57	449.06
390.00	343.82	2615.27	464.40
360.00	316.18	3158.19	483.93
330.00	289.34	3754.18	506.12
300.00	263.01	4404.08	530.16
270.00	236.99	5108.80	555.52

Using the data from the above table a graph has been plotted between average annual cost and period.

Chart 1: Average annual cost vs period



For the above case the replacement has to be made at the ending of 4th period with an average annual cost of 440.90.

CUMULATIVE MAINTENANCE COST WITH MONETARY VALUE ($\Sigma F v^{n-1}$) (8)	SALVAGE VALUE (S) (9)	SALVAGE VALUE WITH MONETARY VALUE (10)
185.46	540	469.57
407.46	510	385.63
650.50	480	315.60
904.32	450	257.29
1161.50	420	208.81
1416.57	390	168.60
1665.56	360	135.34
1905.61	330	107.88
2134.75	300	85.28
2351.70	270	66.74

5.1.2 Case-2: Considering Only Monetary Value

Excluding inflation, average annual cost for each period is attained and using this average annual cost replacement period is found out.

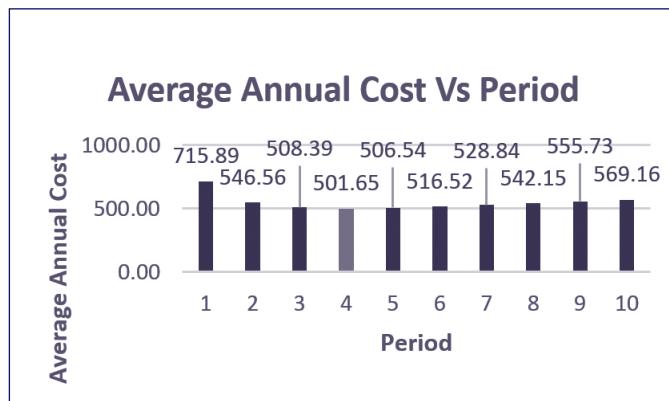
Table 6: Calculation of average annual cost with monetary value.

PERIOD (n) (1)	PRESENT WORTH FACTOR (v) (2)	DISCOUNT FACTOR (v^n) (3)	DISCOUNT FACTOR (v^{n-1}) (4)
1	0.87	0.87	1.00
2	0.87	0.76	0.87
3	0.87	0.66	0.76
4	0.87	0.57	0.66
5	0.87	0.50	0.57
6	0.87	0.43	0.50
7	0.87	0.38	0.43
8	0.87	0.33	0.38
9	0.87	0.28	0.33
10	0.87	0.25	0.28

DIVIDING DISCOUNT FACTOR (Σv^n-1) (5)	MAINTENANCE COST (FORECASTED) (F) (6)	MAINTENANCE COST WITH MONETARY VALUE (F v^n-1) (7)
1	185.46	185.46
1.87	255.30	222.00
2.63	321.42	243.04
3.28	386.03	253.82
3.85	449.80	257.17
4.35	513.04	255.07
4.78	575.92	248.99
5.16	638.54	240.05
5.49	700.95	229.14
5.77	763.20	216.95

TOTAL COST ($TC = C - S * (v^n) + \Sigma F v^n-1$) (11)	AVERAGE COST ($AC = TC / \Sigma v^n-1$) (12)
715.89	715.89
1021.83	546.56
1334.90	508.39
1647.03	501.65
1952.68	506.54
2247.96	516.52
2530.22	528.84
2797.73	542.15
3049.47	555.73
3284.96	569.16

Chart 2: average annual cost vs period



For the above case the replacement has to be made at the ending of 6th period with an average annual cost of 555.26.

5.1.3 Case-3: Without Monetary Value And Inflation

Omitting MONETARY VALUE and inflation, the replacement period is determined based on the average annual cost.

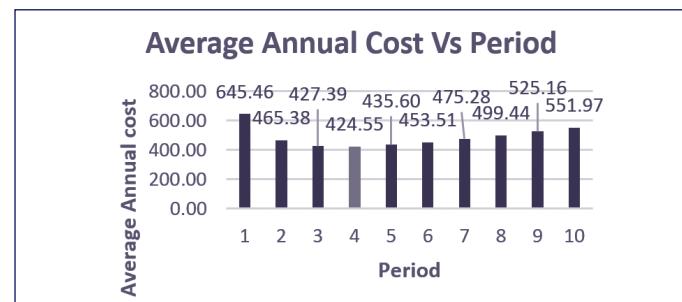
Table 7: Calculation of average annual cost considering no inflation and MONETARY VALUE .

Period (n) (1)	Forecasted Maintenance cost (F) (2)	Salvage value (S) (3)	Cumulative Maintenance Cost (ΣF) (4)
1	185.46	540.00	185.46
2	255.30	510.00	440.76
3	321.42	480.00	762.18
4	386.03	450.00	1148.21
5	449.80	420.00	1598.01
6	513.04	390.00	2111.06
7	575.92	360.00	2686.98
8	638.54	330.00	3325.52
9	700.95	300.00	4026.47
10	763.20	270.00	4789.67

Total annual cost ($T = C - S + \Sigma F$) (5)	Average Annual cost ($AC = TC/n$) (6)
645.46	645.46
930.76	465.38
1282.18	427.39
1698.21	424.55
2178.01	435.60
2721.06	453.51
3326.98	475.28
3995.52	499.44
4726.47	525.16
5519.67	551.97

Using the above data, the graph for average annual cost and period has been plotted.

Chart 3: average annual cost vs period



For the above case the replacement has to be made at the ending of 4th period with an average annual cost of 424.55.

5.2 Replacement Decision Without Salvage Value

5.2.1 Case-4: Considering Monetary Value And Inflation

Taking inflation and monetary value into consideration, average annual cost for each period is acquired and using this the replacement period is found out.

Table 8: Calculation of average annual cost considering monetary value and inflation.

PERIOD (n) (1)	INFLATION (ϕ) (2)	REAL INTEREST RATE ® (3)	PRESENT WORTH FACTOR (v) (4)
1	1.88	4.55	0.96
2	2.33	3.81	0.96
3	2.78	3.24	0.97
4	3.23	2.79	0.97
5	3.67	2.42	0.98
6	4.12	2.12	0.98
7	4.57	1.87	0.98
8	5.02	1.66	0.98
9	5.47	1.47	0.99
10	5.92	1.31	0.99

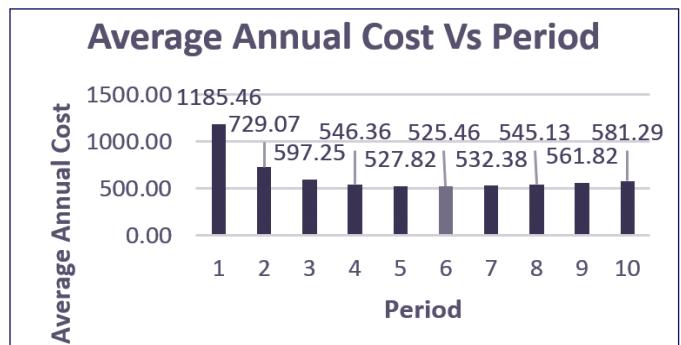
DISCOUNT FACTOR (v^n) (5)	DISCOUNT FACTOR (v^n-1) (6)	DIVIDING DISCOUNT FACTOR (Σv^n-1) (7)
0.96	1.00	1.00
0.93	0.96	1.96
0.91	0.94	2.90
0.90	0.92	3.82
0.89	0.91	4.73
0.88	0.90	5.63
0.88	0.89	6.53
0.88	0.89	7.42
0.88	0.89	8.31
0.88	0.89	9.20

MAINTENANCE COST WITH MONETARY VALUE ($F v^n-1$) (9)	CUMULATIVE MAINTENANCE WITH MONETARY VALUE ($\Sigma F v^n-1$) (10)	TOTAL ANNUAL COST ($TC = C + \Sigma F v^n-1$) (11)
185.46	185.46	1185.46
245.94	431.40	1431.40
301.59	732.99	1732.99
355.48	1088.47	2088.47
408.73	1497.19	2497.19
461.89	1959.08	2959.08
515.29	2474.38	3474.38
569.14	3043.52	4043.52
623.58	3667.10	4667.10
678.70	4345.79	5345.79

AVERAGE ANNUAL COST (AC = TC/ Σv^n-1) (12)
1185.46
729.07
597.25
546.36
527.82
525.46
532.38
545.13
561.82
581.29

Using the above data, a graph has been plotted between average annual cost and period.

Chart 4: average annual cost vs period.



For the above case the replacement has to be made at the ending of 6th period with an average annual cost of 525.46

5.2.2 Case-5: Considering Only Monetary Value

Excluding inflation, average annual cost for each period is attained and using this average annual cost replacement period is found out.

Table 9: Calculation of average annual cost considering only monetary value.

PERIOD (n) (1)	PRESENT WORTH FACTOR (v) (2)	DISCOUNT FACTOR (v^n) (3)	DISCOUNT FACTOR (v^{n-1}) (4)
1	0.87	0.87	1.00
2	0.87	0.76	0.87
3	0.87	0.66	0.76
4	0.87	0.57	0.66
5	0.87	0.50	0.57
6	0.87	0.43	0.50
7	0.87	0.38	0.43
8	0.87	0.33	0.38
9	0.87	0.28	0.33
10	0.87	0.25	0.28

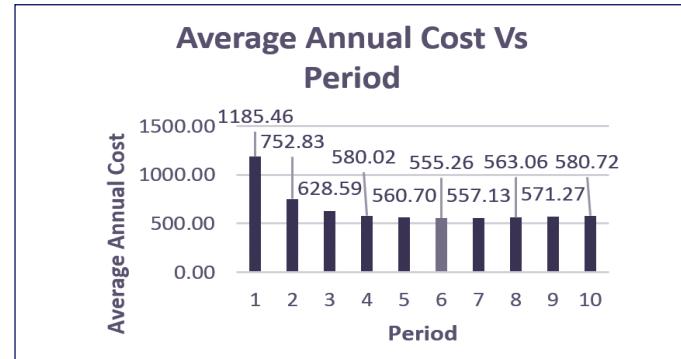
DIVIDEND-G DISCOUNT FACTOR ($\sum v^{n-1}$) (5)	MAINTENANCE COST(FORECASTED) (F) (6)	MAINTENANCE COST WITH MONETARY VALUE (F v^{n-1}) (7)
1	185.46	185.46
1.87	255.30	222.00
2.63	321.42	243.04
3.28	386.03	253.82
3.85	449.80	257.17
4.35	513.04	255.07
4.78	575.92	248.99
5.16	638.54	240.05
5.49	700.95	229.14
5.77	763.20	216.95

CUMULATIVE MAINTENANCE COST WITH MONETARY VALUE ($\sum F v^{n-1}$) (8)	TOTAL COST (TC = $C + \sum F v^{n-1}$) (9)	AVERAGE COST (AC = TC/ $\sum v^{n-1}$) (10)
185.46	1185.46	1185.46
407.46	1407.46	752.83
650.50	1650.50	628.59
904.32	1904.32	580.02
1161.50	2161.50	560.70

1416.57	2416.57	555.26
1665.56	2665.56	557.13
1905.61	2905.61	563.06
2134.75	3134.75	571.27
2351.70	3351.70	580.72

Chart 5: average annual cost vs period.

For the above case the replacement has to be made at the ending



of 6th period with an average annual cost of 555.26.

5.2.3 Case-6: Without Inflation And Monetary Value

Ruling out the inflation and monetary value, the replacement period is attained based on the average annual cost.

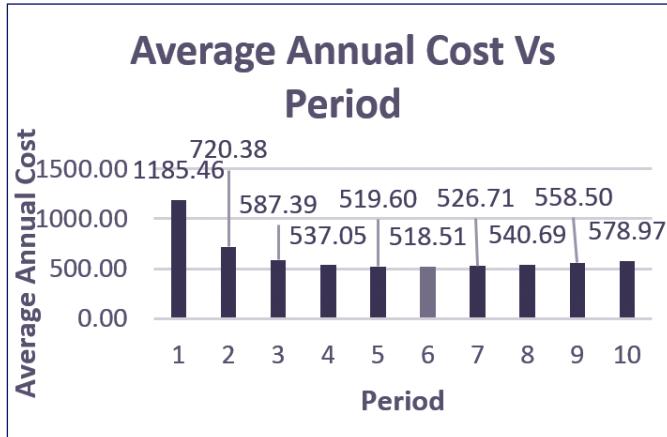
Table 10: Calculation of average annual cost without considering inflation and money value .

PERIOD (n) (1)	MAINTENANCE COST(FORECASTED) (F) (2)	CUMULATIVE MAINTENANCE COST ($\sum F$) (3)
1	185.46	185.46
2	255.30	440.76
3	321.42	762.18
4	386.03	1148.21
5	449.80	1598.01
6	513.04	2111.06
7	575.92	2686.98
8	638.54	3325.52
9	700.95	4026.47
10	763.20	4789.67

TOTAL ANNUAL COST ($TC = C + \Sigma F$) (4)	AVERAGE ANNUAL COST ($AC = TC/n$) (5)
1185.46	1185.46
1440.76	720.38
1762.18	587.39
2148.21	537.05
2598.01	519.60
3111.06	518.51
3686.98	526.71
4325.52	540.69
5026.47	558.50
5789.67	578.97

The data from the previous table is used to plot a graph between average annual cost and period.

Chart 6: average annual cost vs period



For the above case the replacement has to be made at the ending of 6th period with an average annual cost of 518.51.

6 Analogy Of All The Case Studies

Using the above data, required comparisons are made and certain conclusions are drawn from them.

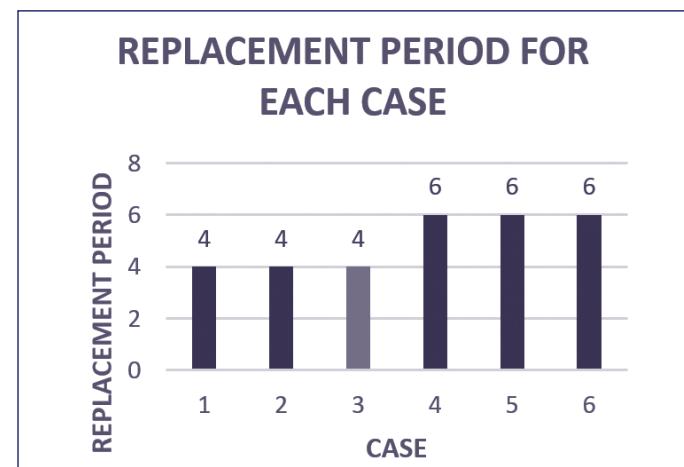
The comparisons made are:

6.1 Cases Vs Replacement Period

Table 11: replacement period for different cases.

CASE	PERIOD
1	4
2	4
3	4
4	6
5	6
6	6

Chart 7: Comparison graph for replacement period.

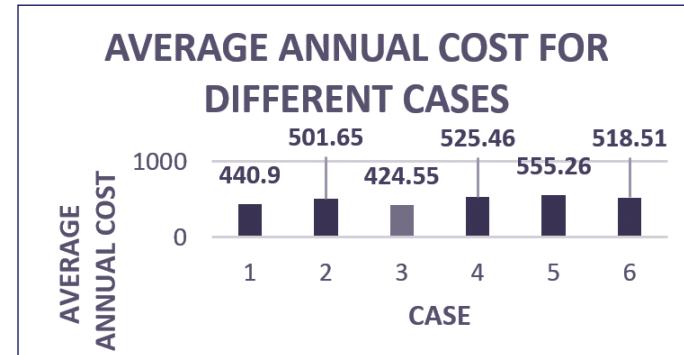


6.2 Case Vs Average Annual Cost

Table 12: Average annual cost for different cases.

CASE	AVERAGE ANNUAL COST
1	440.90
2	501.65
3	424.55
4	525.46
5	555.26
6	518.51

Chart 8: Comparison graph for average annual cost for different cases



CONCLUSION

It's evident that the replacement period was either 4 or 6 for all the cases that were considered but the average annual cost varied. It is conspicuous that the least cost can be obtained only when there is no monetary value or inflation and which, in this case, is the third case where only salvage value is considered. The least average annual cost was 424.55 rupees obtained when inflation is excluded while the highest average annual cost was 555.26 rupees when both inflation and salvage value are excluded showing the effect of inflation and monetary value on the cutting tool. This study gives a different perspective of cost optimization and estimation of optimal replacement time of cutting tools in a manufacturing workshop.

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AUTHORS

Revanth Sai Yayavaram, UG student, Department of Mechanical Engineering, MVSR Engineering College, Hyderabad, India

Dr. S. Gajanana, Professor & Dean, MVSR Engineering College, Hyderabad, India

Kshetty Rahul, UG student, Department of Mechanical Engineering, MVSR Engineering College, Hyderabad, India